Quantitative & Data Reasoning Companion

Unit Cluster: Topic 4 Water Systems & Soil Carbon Fluxes · Upright 11–16 pt Sans-Serif Layout · Lining Tabular Numerals

Four-Anchor Method Used Throughout This Workbook

Every calculation separates reading comprehension from arithmetic execution using four invariant anchors so every student tracks unit conversions reliably.

Anchor	Step Role	What to Check Before Writing
▲ Target Quantity	Command term and output unit	Underline exact unit required in answer (e.g. mm yr^{-1}, %, km^3).
• Given Data	Extract values and matching units	List numerator and denominator values explicitly with source units.
■ Method & Formula	Select relationship and substitute	Confirm signs (+ gain / - loss) and convert unlike prefixes first.
◆ Check & Result	Round clearly and inspect magnitude	Check significant figures match data and verify direction of change.

Why This Format Supports Diverse Learners

100% upright typography, generous white space, maximum 65 characters per line, left-aligned step blocks, and explicit conservation equations eliminate visual crowding without reducing IB Diploma assessment rigour.

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Skill 1: Catchment Mass Balance & Storage Change

Scenario (Topic 4.1 Hydrological Cycle): A temperate river basin receives mean annual precipitation P = 1,240 mm yr^{-1}. Measured surface runoff R = 510 mm yr^{-1} and deep groundwater percolation G = 165 mm yr^{-1}. Assuming long-term steady storage (ΔS = 0), calculate mean annual evapotranspiration (ET) in mm yr^{-1}.

▲ **Target:** Mean annual evapotranspiration (ET) in **mm yr^{-1}**.

• **Given:** P = 1,240 mm yr^{-1} (input) | R = 510 mm yr^{-1} (output) | G = 165 mm yr^{-1} (output) | ΔS = 0.

■ **Equation:** P = R + ET + G + ΔS → ET = P - (R + G) = 1,240 - (510 + 165) = 1,240 - 675.

◆ **Result:** ET = **565 mm yr^{-1}**. Check: 510 + 565 + 165 = 1,240 mm yr^{-1} balanced.

Guided Practice (Student Work Area)

A Mediterranean sub-catchment records P = 680 mm yr^{-1}, ET = 415 mm yr^{-1}, and R = 190 mm yr^{-1}. Calculate aquifer recharge G assuming ΔS = 0.

Skill 2: Runoff Coefficient, Infiltration Shift & Lag Time

Hydrology Ratio & Directional Rail

Ratio Formula	Direction Check on Land-Use Urbanization
Runoff Ratio C = Q_{runoff} / P_{rain}	Impermeable tarmac ↑ → Infiltration ↓ → Runoff Ratio C ↑ ([+] direct rise)
% Runoff Change = ((Q_2 - Q_1) / Q_1) × 100	Basin Lag Time ↓ → Peak Stream Discharge Q_{max} ↑ ([-] inverse shift)

Worked Problem (Storm Hydrograph Audit): Before impermeable paving, a 4.0 km^2 basin produced Q_1 = 18.0 m^3 s^{-1} peak storm discharge during a 50 mm rain event. Following commercial development, peak discharge rose to Q_2 = 29.7 m^3 s^{-1} under the identical storm input. Calculate the percentage increase in peak discharge.

▲ **Target:** Percentage change (%) in peak river discharge Q_{max}.

• **Given:** Initial peak Q_1 = 18.0 m^3 s^{-1} | Urbanized peak Q_2 = 29.7 m^3 s^{-1}.

■ **Substitution:** ((29.7 - 18.0) / 18.0) × 100 = (11.7 / 18.0) × 100 = 0.65 × 100.

◆ **Result: +65.0% increase** in peak discharge (inverse decrease in infiltration residence time).

Skill 3: Turnover Time & Groundwater Extraction Budget

Formula Box: Residence Time τ (yr) = Reservoir Storage Volume (km^3) / Annual Flow Rate ($km^3\ yr^{-1}$).

Aquifer Compartment	Storage Volume (km^3)	Natural Recharge Flux ($km^3\ yr^{-1}$)	Human Pumping ($km^3\ yr^{-1}$)
Shallow Alluvial Lens	**14.4**	**0.48**	**0.40**
Confined Sandstone Basin	**420.0**	**0.35**	**1.75**

▲ **Target:** Natural turnover time (τ) for both aquifers in years, plus net annual depletion rate of the Confined Basin.

• **Given:** Shallow V = 14.4 km^3, Flux = 0.48 $km^3\ yr^{-1}$ | Confined V = 420.0 km^3, Recharge = 0.35 $km^3\ yr^{-1}$, Extraction = 1.75 $km^3\ yr^{-1}$.

■ **Calculate:** Shallow τ = 14.4 / 0.48 = 30 yr. Confined τ = 420.0 / 0.35 = 1,200 yr. Net Confined Flux = 0.35 - 1.75 = -1.40 $km^3\ yr^{-1}$.

◆ **Synthesis:** Confined extraction exceeds natural recharge by 5:1, classifying it as non-renewable groundwater mining.

Instructor Note: Aligns directly with IB ESS Topic 4.2 Sustainable Water Management & Exam Calculator Accuracy.

Worked Verification Key (For Educators & Learning Support Staff)

Problem	Exact Substituted Numerator / Denominator	Verified Answer	Common Error Check
Skill 1 Practice	G = 680 - (415 + 190) = 680 - 605	**75 mm yr^{-1}**	Adding rather than subtracting ET removes input mass conservation.
Skill 2 Hydrograph	(29.7 - 18.0) / 18.0 × 100	**+65.0%**	Dividing by final discharge Q_2 (29.7) yields 39.4% (incorrect base).
Skill 3 Exhaustion Check	Remaining span at -1.40 km^3 yr^{-1}: 420 / 1.40	**300 years**	Confusing natural turnover (1,200 yr) with pumped depletion span (300 yr).

Complete Pillar 12 Quantitative Track Inclusions

When added to your campus licence, all 11 IB ESS units include editable Word (.docx) and PDF Quantitative companions plus Profile 1 (Memory & Reading), Profile 2 (Structured Focus), and the 3-Part Staff & Teaching Assistant Briefing Pack.

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